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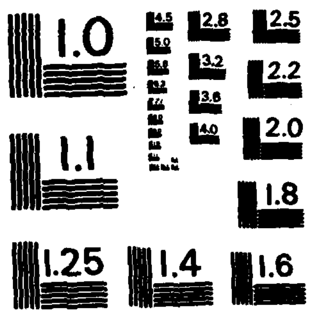

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EVOLUTION OF NONLINEAR WAVE GROUPS ON WATER OF SLOWLY-VARYING DEPTH

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Work done during the report period

The Fourth periodical report concentrated on the numerical solution of N.L.S. equation with varying coefficients.

The numerical treatmant is based on the analytical solution developed in the third periodical report. The solution is valid using periodic boundaries and a very slow variation of the bottom compared with the period's length. The solution was compared with other numerical schemes and yield satisfactory results.

All technical details are presented in the following paper.

EVOLUTION OF AN UNSTABLE WATER WAVE TRAIN MOVING OVER A SLOWLY VARYING BOTTOM

(Part II)

by R. Iusim and Y. Frostig.

Introduction

The aim of this paper is to present quantitative information about the approximate solution of the non-linear Schroedinger ~~(N.L.S.)~~ equation with varying coefficients and periodic boundary conditions ~~developed in the previous report (2)~~, and comparing it with numerical solutions.

In section I we present a brief review of the method is developed and ~~in (2), which~~ is based in the evaluation of an "initial disturbance" at every point, and local application of the analytical solution ~~given by M. Stiassnie et al. (1).~~ In section ~~II we~~ provide quantitative information about the variation of the local "initial disturbance" is provided and ~~in section III~~ we present the comparison of the solution ~~developed in (2)~~ with numerical solutions.

I. Review of the Method Developed in the Previous Report

The problem to be solved is:

$$i\psi_X + \psi_{TT} + \mu(X)|\psi|^2\psi = 0 \tag{1.1}$$

with the boundary condition

$$\psi\Big|_{x=0} = 1 + 2\beta e^{i\alpha}\cos(2\pi T) \tag{1.2}$$

Following Stiassnie and Kroszynsky (1) we approximate the solution of (1.1), (1.2) by the truncated Fourier series:

$$\psi^1(T,X) = \sum_{n=-1}^{1} D_n(X)e^{2\pi inT} \qquad (1.3)$$

where $D_n(X)$ is the complex Fourier coefficient of modulus $|D_n|$ and argument α_n:

$$D_n = |D_n|e^{i\alpha_n} \qquad (1.4)$$

The boundary condition at $X = 0$ implies that

$$|D_o(o)| = 1, \qquad \alpha_o(o) = 0$$
$$(1.5)$$
$$|D_1(o)| = \beta, \qquad \alpha_1(o) = \alpha$$

From substitution of (1.3) into the N.L.S. (1.1) the following set of equations is obtained

$$i\frac{dD_o}{\partial X} + \mu(X)\ [(|D_o|^2 + 4|D_1|^2)D_o + 2D_1^2D_o^*] = 0$$
$$(1.6)$$
$$i\frac{dD_{\pm 1}}{dX} + \mu(X)\ [(2|D_o|^2 + 3|D_1|^2 - \tfrac{1}{2}P)D_1 + D_o^2D_1^*] = 0$$

where $P = 8\pi^2/\mu$. The range of variation of P is $1 < P < 4$. (see Stiassnie, et al. (1)).

Assuming that the bottom varies so slowly that it may be considered "locally constant", then the solution given by Stiassnie et al. (1) for constant depth can be applied locally.

This solution is periodic in X. Assuming that the bottom is "locally constant" means that it is nearly constant in the period of the solution.

Defining:

$$Z(X) = 2|D_1(X)|^2 \tag{1.7}$$

the exact solution of system (1.6) is given by the set

$$|D_o|^2 + Z = 1 + 2\beta^2 \tag{1.8}$$

$$\cos 2(\alpha_o - \alpha_1) = \frac{I(P) + 3/2\, Z^2 + [P - 2(1+2\beta^2)]Z}{2Z(1+2\beta^2 - Z)} \tag{1.9}$$

$$Z(X) = \frac{aQ-b}{Q-1} \tag{1.10}$$

$$Q = \frac{b - \varepsilon(P)}{a - \varepsilon(P)} \left[\frac{1 - \frac{(a-b)}{(a-\varepsilon(P))} \left(\frac{(\varepsilon(P)-c)(\varepsilon(P)-d)}{(b-c)(b-d)}\right)^{1/2} \frac{sd}{cn}}{1 + \frac{(a-b)}{(a-\varepsilon(P))} \frac{(a-d)}{(b-d)} \frac{(b-c)}{(a-c)}\, sd^2} \right] cd^2 \tag{1.11}$$

where $cd = \frac{cn}{dn}$ **and** $sd = \frac{sn}{cn}$ **represent the Jacobian elliptic functions of argument**

$$y = -\tfrac{1}{4}(7ab)^{1/2}\, \mu X \tag{1.12}$$

and modulus k given by

$$k^2 = 1 - \frac{(a-b)(c-d)}{(a-c)(b-d)} \tag{1.13}$$

$$a = P\{1 + [1 + \frac{2I(P)}{P^2}]^{1/2}\} \tag{1.14}$$

$$b = \frac{1}{7}[4(1+2\beta^2) - P]\,\{1 + [1 - \frac{14 I(P)}{4(1+2\beta^2) - P}]^{1/2}\} \tag{1.15}$$

$$c = \frac{1}{7}\left[4(1+2\beta^2)-P\right]\left\{1-\left[1-\frac{14I(P)}{4(1+2\beta^2)-P}\right]^{\frac{1}{2}}\right\} \qquad (1.16)$$

$$d = P\left\{1-\left[1+\frac{2I(P)}{P^2}\right]^{\frac{1}{2}}\right\} \qquad (1.17)$$

if $I(P) > 0$ or vice-versa, d given by (1.16) and c by (1.17) if $I(P) < 0$.

The approximate solution depends on two parameters: $I(P)$ and the local initial condition for Z: $\varepsilon(P)$. Given $I(P)$ and $\varepsilon(P)$, $Z(X)$ is determined from (1.8) to (1.17). At the case of constant depth:

$$\mathcal{E}(P) \equiv \text{const} = 2\beta^2 \qquad (1.18)$$

and $$I(P) = \text{const} = \beta^2\left[2\beta^2 + 4(1+\cos 2\alpha) - 2P\right] \qquad (1.19)$$

Under the assumption that the bottom is "locally constant" in the sense previously mentioned, we derived in (2) the following approximate differential equation for $I(P)$.

$$\frac{\partial I(P)}{\partial P} = -\sqrt{\frac{P}{7}}\,(4-P)\cdot\frac{2\ln\left(\dfrac{1+\sqrt{\frac{(4-P)}{7P}}}{1-\sqrt{\frac{(4-P)}{7P}}}\right)}{\ln\left|\dfrac{2P^2(4-P)^2}{(2P-1)I(P)}\right|} \qquad (1.20)$$

with the initial condition (1.19).

Equation (1.20) can be easily solved numerically.

In order to derive equation (1.20) we assumed in that the initial condition of $Z(X)$ for local disturbance satisfies the condition

$\varepsilon = \varepsilon(P) << 1$. We have chosen as the local initial condition $\varepsilon(P)$, the minimal positive value of the oscilatory function $Z(X)$, The minimal value of $Z(X)$ is found by equalling to zero its derivative given by

$$\frac{\partial Z}{\partial X} = \frac{1}{\mu}\sqrt{[2Z(1+2\beta^2-Z)]^2-[\tfrac{3}{2}Z^2+[P-2(1+2\beta^2)]Z - I(P)]^2} \qquad (1.21)$$

(see Stiassnie et al. (1)).

If $I(P) > 0$ the minimal positive value of Z is given by

$$\varepsilon(P) = \frac{1}{7}\,[4(1+2\beta^2)-P]\;\{1-[\,1-\frac{14I(P)}{4(1+2\beta^2)-P}\,]^{\frac{1}{2}}\} \equiv c \qquad (1.22)$$

and from (1.9) it can be seen that it corresponds to $\cos 2(\alpha_o-\alpha_1) = 1$

If $I(P) < 0$ the minimal positive value of Z is given by

$$\varepsilon(P) = P\,\{\,1-[\,1+\frac{2I(P)}{P^2}]^{\frac{1}{2}}\} \equiv c \qquad (1.23)$$

and it corresponds to $\cos 2\,(\alpha_o-\alpha_1) = -1$.

The approximate wave envelope $\psi_1(T,X)$ is given by

$$|\psi^1|^2 = \{-2I(P) + 2[4(1+2\beta^2)-P]Z - 7Z^2 +$$

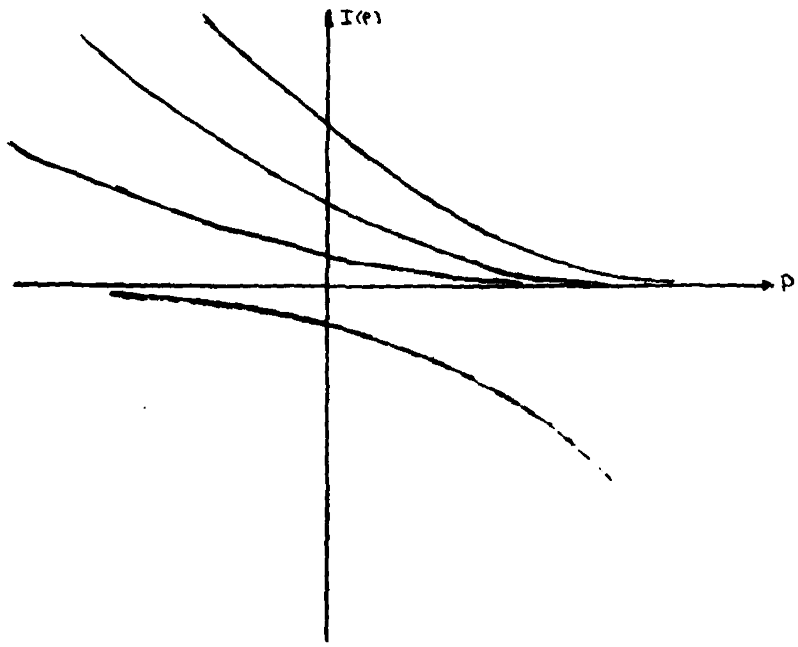
$$+ [S\,(2I(P) + 2PZ-Z^2)^{\frac{1}{2}} + 4Z\cos 2\pi T]^2\}\,/8Z. \qquad (1.24)$$

where S is the sign of $\cos(\alpha_1-\alpha_o)$.

II. The behavior of I(P)

The right hand side of equation (1.20) is always negative for the range of values of P considerated $1 < P < 4$, yielding that I(P) is a decreasing function of P. On the other hand, it can be easily seen that $I(P) \equiv 0$ *is a solution* of equation (1.20). That means that if the initial condition $I(P_o)$ is positive, the solution remains positive, and asimptotic to the horizontal axis. If the initial condition is negative, the solution remains negative. The *expected* behavior of the solution is given in figure. 1.

Figure 1



If $I(P) = 0$ then eqs. (1.16) and (1.17) yield $c \equiv d \equiv 0$ and from (1.13) we obtain $k \equiv 1$. Then the period of the elliptic functions in (1.11) is infinite (see (3)).

The solution $Z(X)$ in this case is given by (1.10) where

$$Q = \left[\frac{1 - \left(\frac{a-b}{a-\varepsilon} \cdot \frac{\varepsilon}{b}\right) \frac{\mathrm{tgh}\, y}{\mathrm{sech}^2 y}}{1 + \left(\frac{a-b}{a-\varepsilon} \cdot \frac{\varepsilon}{b}\right) \frac{\mathrm{tgh}^2 y}{\mathrm{sech}^2 y}} \right] \tag{2.1}$$

where a and b are given by (1.14) and (1.15) respectively. From (1.10) and (2.1) it can be seen that $Z(X)=0(\varepsilon)$. This solution is unstable to disturbances in the parameter $I(P)$, giving rise to the oscillatory solution (1.10),(1.11) which grows from its minimal value $Z = c = 0(\varepsilon)$, to its maximal value $z = b$.

For this reason all the numerical calculations are unstable near the value $I(P) = 0$. On the other hand, when $I(P)$ tends to zero, the period of the oscillatory solution $Z(X)$ tends to infinity, and then our assumption that the bottom is nearly constant in a period of the solution does not hold any more.

These reasons impose new restrictions in P. The permissible range of variation of P must ensure the conditions $|I(P)| >> 1$, but $I(P)$ not nearly zero.

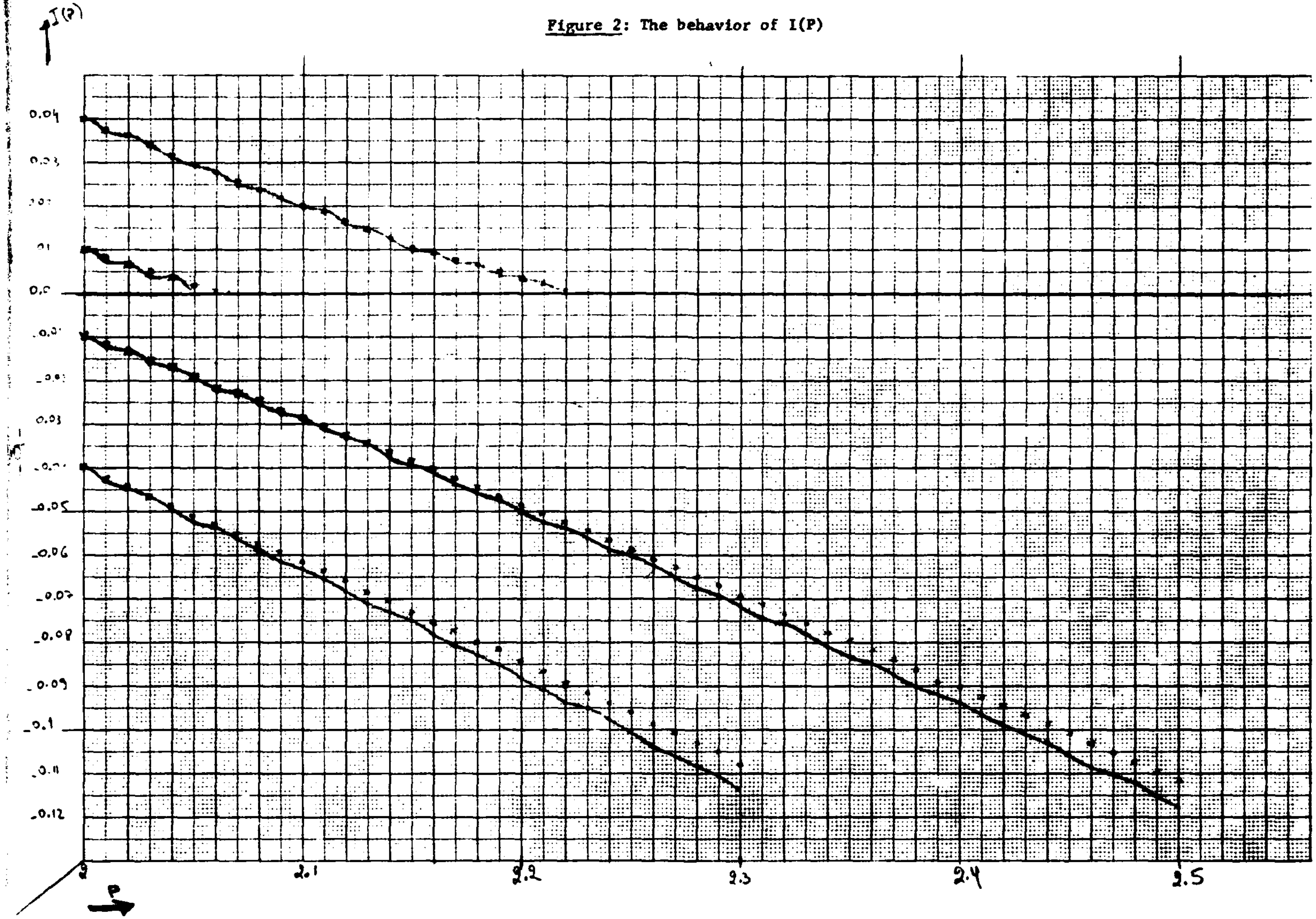
From the numerical results we actually impose the condition $|I(P)| > 0(10^{-3})$.

The solution of equation (1.20) is compared with the one obtained by solving numerically the set (1.6) and using the equality

$$I(P) = |D_1|^2 \cdot [2|D_1|^2 + 4|D_o|^2 - 2P + 4|D_o|^2 \cos 2(\alpha_1 - \alpha_o)] \qquad (2.2)$$

Equation (1.16) was solved by means of the second order Runge Kutta method, and by the trapezoidal rule, obtaining identical results. System (1.6) was solved by the trapezoidal rule. In both cases we considered the simple case where P is linear on X: $P = P_o + 0.1X$; $P_o = 2$ starting at $X = 0$. The results are presented in figure 2. Full lines represent the solutions obtained by (1.6) and (2.2), while dotted lines are the results from numerical solutions of equation (1.20).

Figure 2: The behavior of I(P)



III. Comparison between the solution presented in section I and numerical solutions

In order to appraise the validity of the behavior predicted by equation (1.20) and the analytical solution (1.7) to (1.17), we compare it with a reference solution obtained by means of numerical schemes. In the first place we compare it with the numerical solution of the set (1.6) obtained by means of the trapezoidal rule. Figures 3 and 4 show the wave envelope $|\psi(0,X)|$ for the case $P = P_o + 0.1X$, $P_o = 2$, $\alpha = 0$, $\beta = 0.1$.

These values correspond to the initial condition $I(P_o) = 0.0402$. Full lines represent the solution of the set (1.6) while dotted lines are the results obtained by solving equation (1.20) and applying (1.22), (1.11) to (1.17) and (1.24) at $P = 2$, $P = 0.5$ and $P = 2.16$. Figures 5 and 6 represent the wave envelope $|\psi(0,T)|$ when $\alpha = \pi/2$, $\beta = 0.1$ $P = P_o+0.1X$, $P_o = 2$, $(I(P_o) = -0.0998)$. The full lines represent the solution of set (1.6), and the dotted ones the solution given by (1.20), (1.23), (1.10) to (1.17) and (1.24) at $P = 2$ and $P = 2.5$. In both cases a very satisfactory correspondence of the wave envelop's shape is observed.

On the other hand, the solution of set (1.6) is compared to the numerical solution of the N.L.S. equation when $P = 2 + 0.1X$. In order to solve the N.L.S. equation two independent alternative numerical schemes were employed, obtaining identical results. The first one is the Crank Nicholson scheme

$$i\,\frac{\psi_j^{n+1}-\psi_j^n}{\Delta X}+\tfrac{1}{2}\left(\frac{\psi_{j-1}^{n+1}-2\psi_j^{n+1}+\psi_{j+1}^{n+1}}{(\Delta T)^2}+\frac{\psi_{j-1}^{n}-2\psi_j^{n}+\psi_{j+1}^{n}}{(\Delta T)^2}\right)$$

$$+\ \mu^{n+\frac{1}{2}}\ |\psi_j^n|\cdot|\psi_j^{n+1}|\cdot\frac{(\psi_j^{n+1}+\psi_j^n)}{2} = 0 \qquad (3.1)$$

where $\psi_j^n = \psi(T_j,X_n)$, $\mu^n = \mu(X_n)$, $T_j = (j-1)\Delta T$, $X_n = n\Delta X$, ΔT are J-1 equal segments that span the interval $0 < T < \frac{1}{2}$ and ΔX are equal intervals that span the X coordinate. The scheme is subject to the initial condition (1.2) and to the boundary conditions $\psi_o^n = \psi_2^n$; $\psi_{J+1}^n = J_{J-1}^n$.

In the second approach we seek an approximate solution of the form

$$\psi^N(T,X) = \sum_{n=-(N-1)}^{N} D_n(X)e^{2\pi i n T} \qquad (3.2)$$

Given the set $\psi^N(T_j,X)$ $j = -(N-1),\ldots,0,1,\ldots N$, the corresponding Fourier coefficients $D(x)_{-(N-1)},\ldots,\ D_o(x)\ ,\ldots,D_N(x)$ are found by means of the Fast Fourier Transform, and conversely, given $D_{(N-1)}(x),\ldots,$ $D_o,\ldots,D_N(x)$ the set $\psi^N(T_j,X)j=-(N-1),\ldots,0,1,\ldots,N$ is found by means of the Inverse Fourier Transform.

Substitution of (3.2) in the N.L.S. equation (1.1) yields:

$$\frac{idD_n}{dX} + i(2\pi n)^2 D_n - i\mu A_n = 0 \qquad (3.3)$$

$$D_{-n} = D_n \qquad (n = 0,1,\ldots,N)$$

where $A_n = A_n(X)$, $n = -(N-1),\ldots,0,\ldots,N$ are the Fourier coefficients of the set

$$|\psi^N(T_j,X)|^2\cdot\psi^N(T_j,X),\quad j = -(N-1),\ldots,0,\ldots N\ .$$

The solution of (3.1) is given by

$$D_{\pm n}(x) = [D_n(0) + i\mu \int_o^x A_n(t)e^{i(2\pi n)^2 t}dt]\ e^{-i(2\pi i)^2 x} \qquad (3.4)$$

which can be rewritten in the form

$$D_{\pm n}(x+\Delta X) = [D_n(x) + i\mu e^{-i(2\pi n)^2 x} \int_x^{x+\Delta x} e^{i(2\pi n)^2 t} A_n(t)dt]\ e^{-i(2\pi n)^2 \Delta X}$$

$$n = 0,1,\ldots,N \qquad (3.5)$$

Assuming that the chosen increment ΔX is sufficiently small, the integral in (3.5) is computed approximately as

$$\tfrac{1}{2}[A_n(x) + A_n(x+\Delta X)] \int_x^{x+\Delta x} e^{i(2\pi n)^2 t}dt \qquad (3.6)$$

Substitution of (3.6) into (3.5) leads to the numerical scheme

$$D_o(X+\Delta X) = D_o(X) + \Delta X i\mu (A_o(X+\Delta X) + A_o(X))/2$$

$$(3.7)$$

$$D_{\pm n}(X+\Delta X) = f_n D_n(X) + (1-f_n)(A_n(X+\Delta X)+A_n(X))/(P\cdot n^2)$$

$$n = 1,2,\ldots,N$$

where $f_n = e^{-i(2\pi n)^2 \Delta X}$

Starting with the known values of D_n at level X, we proceed to compute $A_n(X)$ by means of the Fast Fourier Transform. We use this value as a first guess for $A_n(X+\Delta X)$ and obtain an estimate for $D_n(X+\Delta X)$ from (3.7). The Fast Fourier Transform and expression (3.7) are then applied iteratively until no change between two successive estimates are detected. The process is then advanced a further step in X. The first step uses as starting values the initial condition (1.2).

Figures 7 and 8 show the solution of the N.L.S. equation when $P = P_o + 0.1X$, $P_o = 2$, for the initial conditions $\alpha = 0$, $\beta = 0.1$ ($I(P_o) = 0.0402$) and $\alpha = 0$, $\beta = 0.05$ ($I(P_o) = 0.01000125$) respectively, while in figures 9 and 10 it can be seen the solution for the initial conditions $\alpha = \pi/2$, $\beta = 0.1$ ($I(P_o) = -0.0998$), Full lines in these figures represent the solution of the N.L.S. equation, while dotted lines represent the solution of system (1.6).

Figure 3: The wave envelope $|\psi(o,x)|$.

$P = 2+0.1X$, $\alpha = 0$, $\beta = 0.1$.

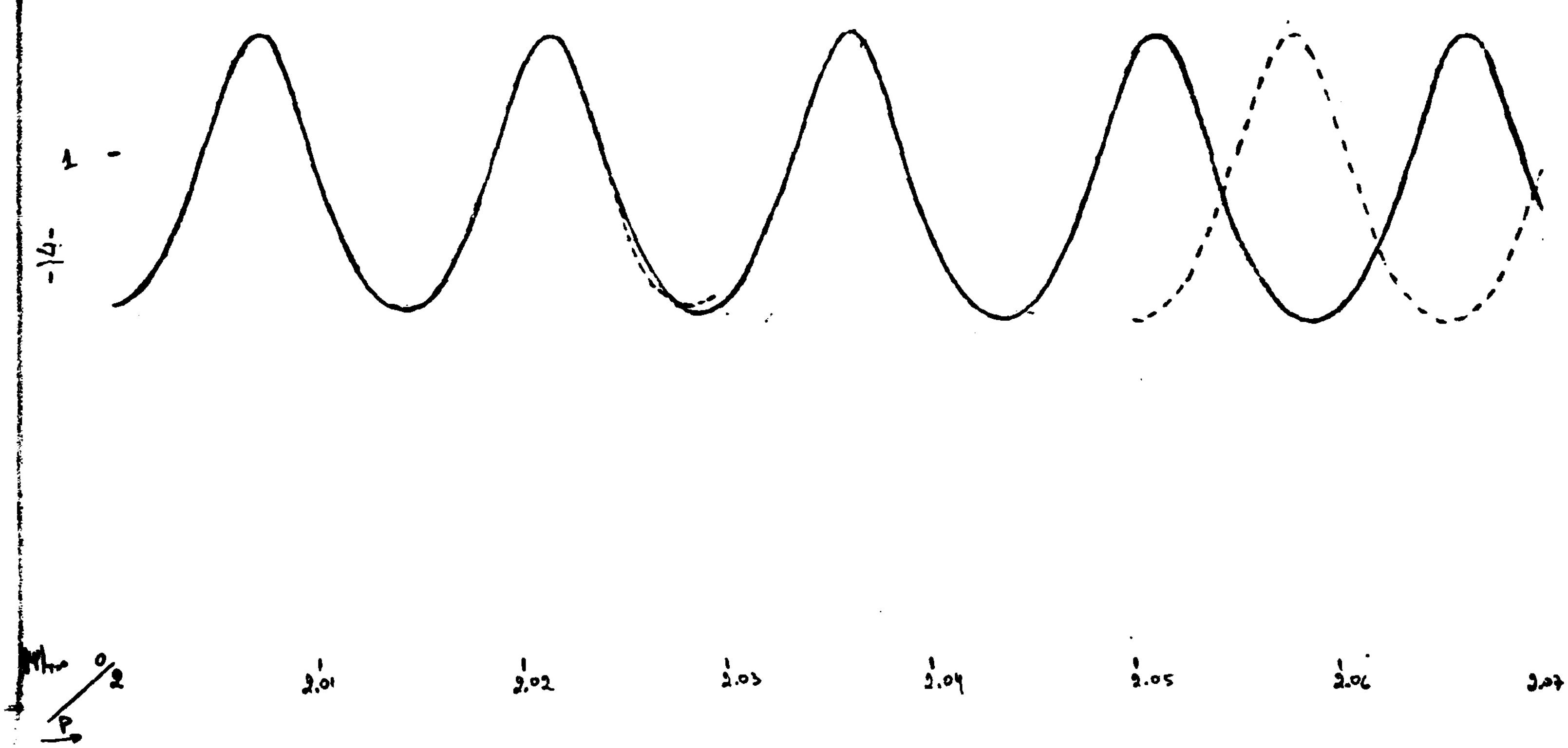
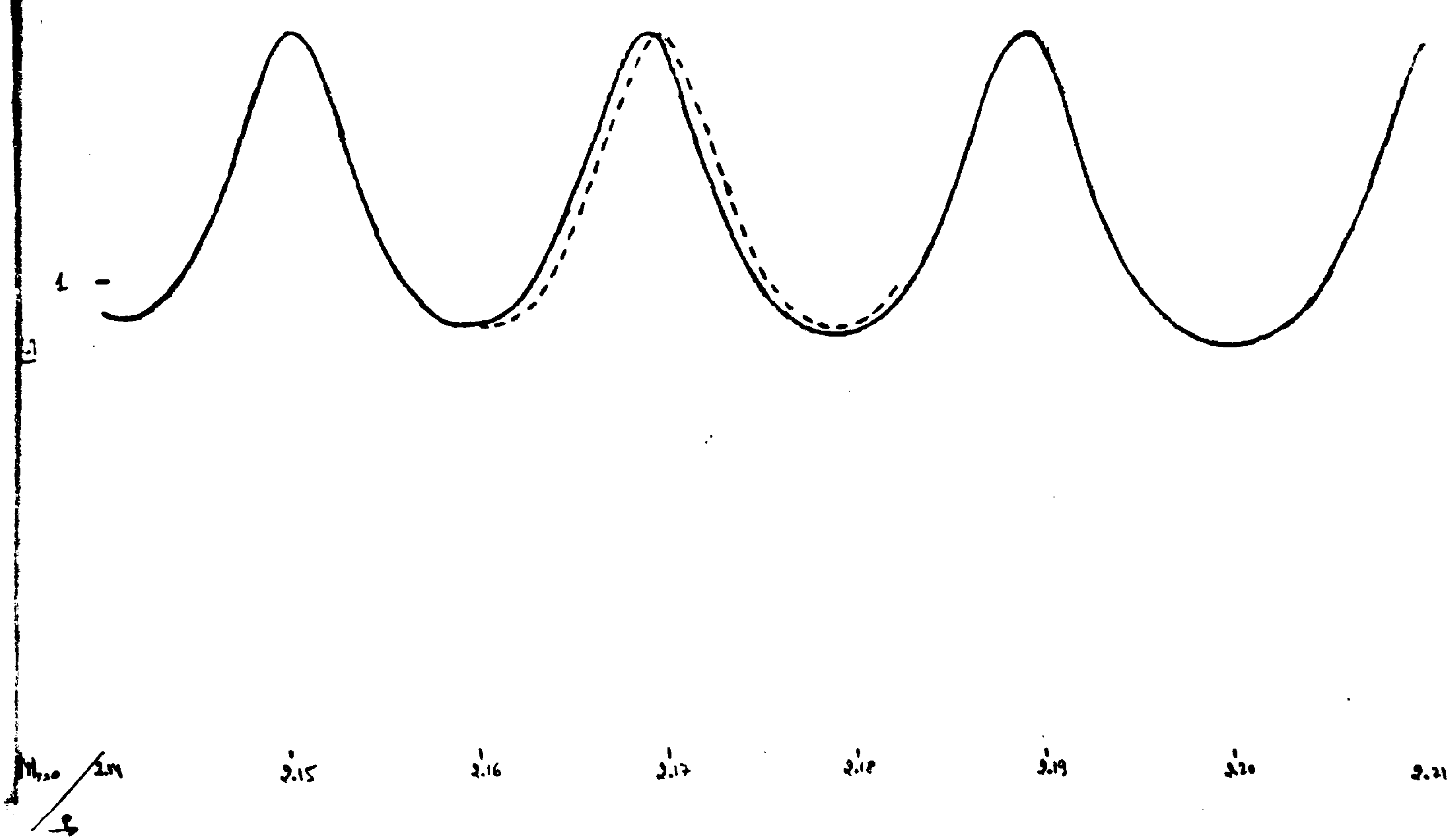


Figure 4: The wave envelope $|\psi(o,x)|$

$P = 2+0.1X, \quad \alpha = 0, \quad \beta = 0.1.$



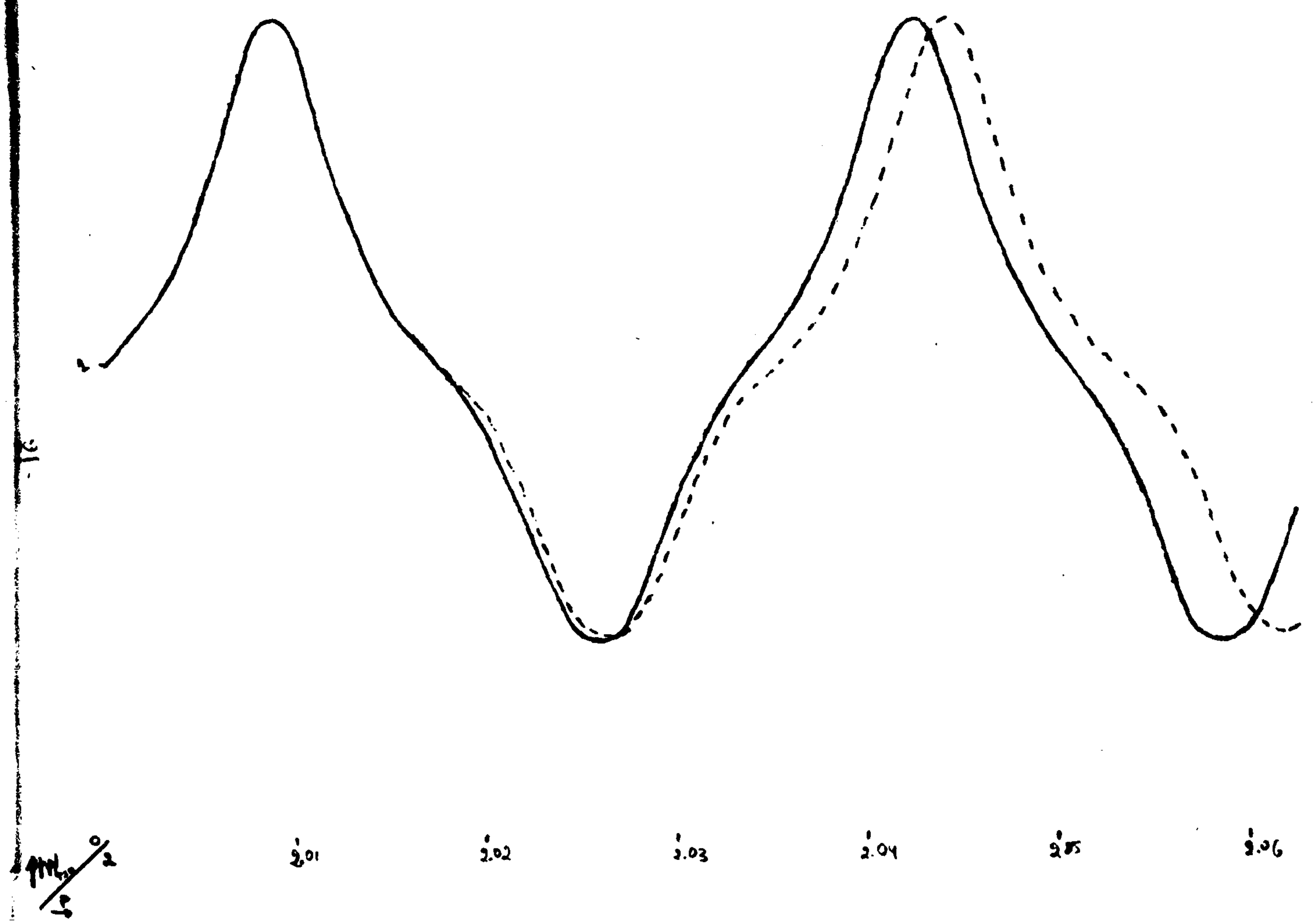


Figure 5: The wave envelope $|\psi(o,x)|$

$P = 2 + 0.1X$, $\alpha = \pi/2$, $\beta = 0.1$.

Figure 6: The wave envelope $|\psi(o,x)|$

$P = 2+0.1X$, $\alpha = \pi/2$, $\beta = 0.1$.

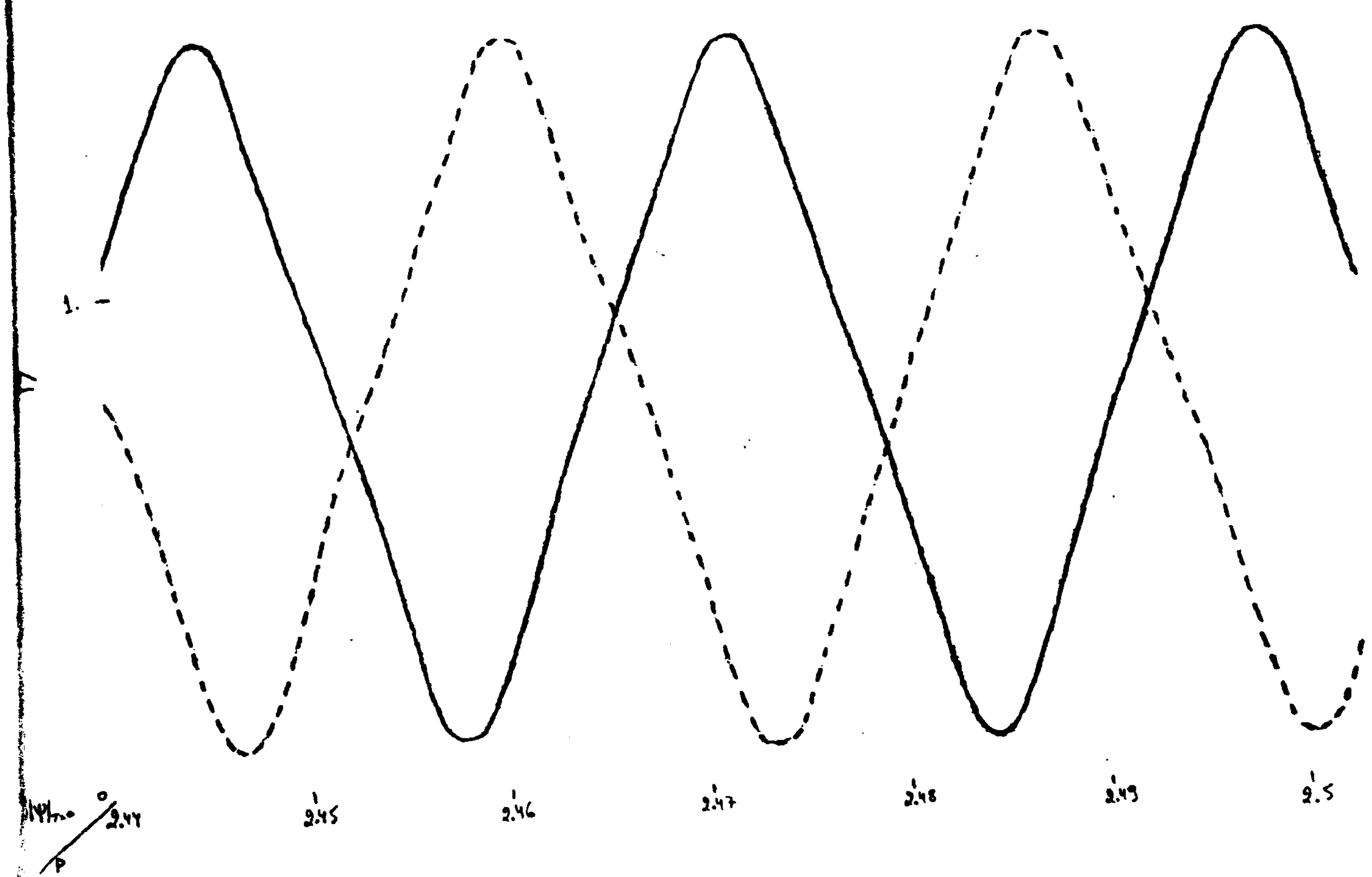


Figure 7: The wave envelope $|\psi(o,x)|$

$P=2+0.1X$, $\alpha=0$, $\beta=0.1$

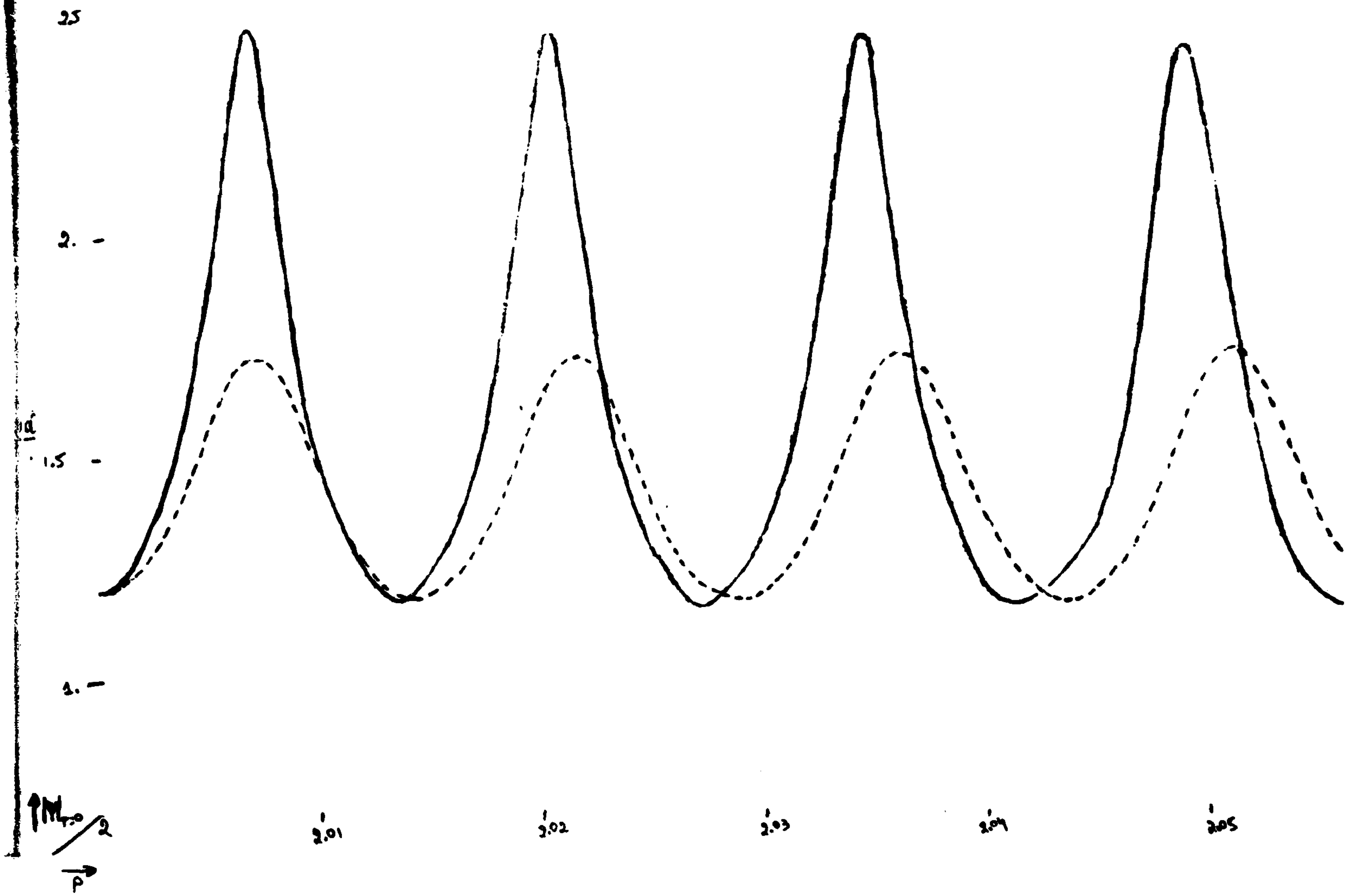


Figure 8: The wave envelope $|\psi(o,x)|$

$P=2+0.1X, \quad \alpha = 0, \quad \beta = 0.05$

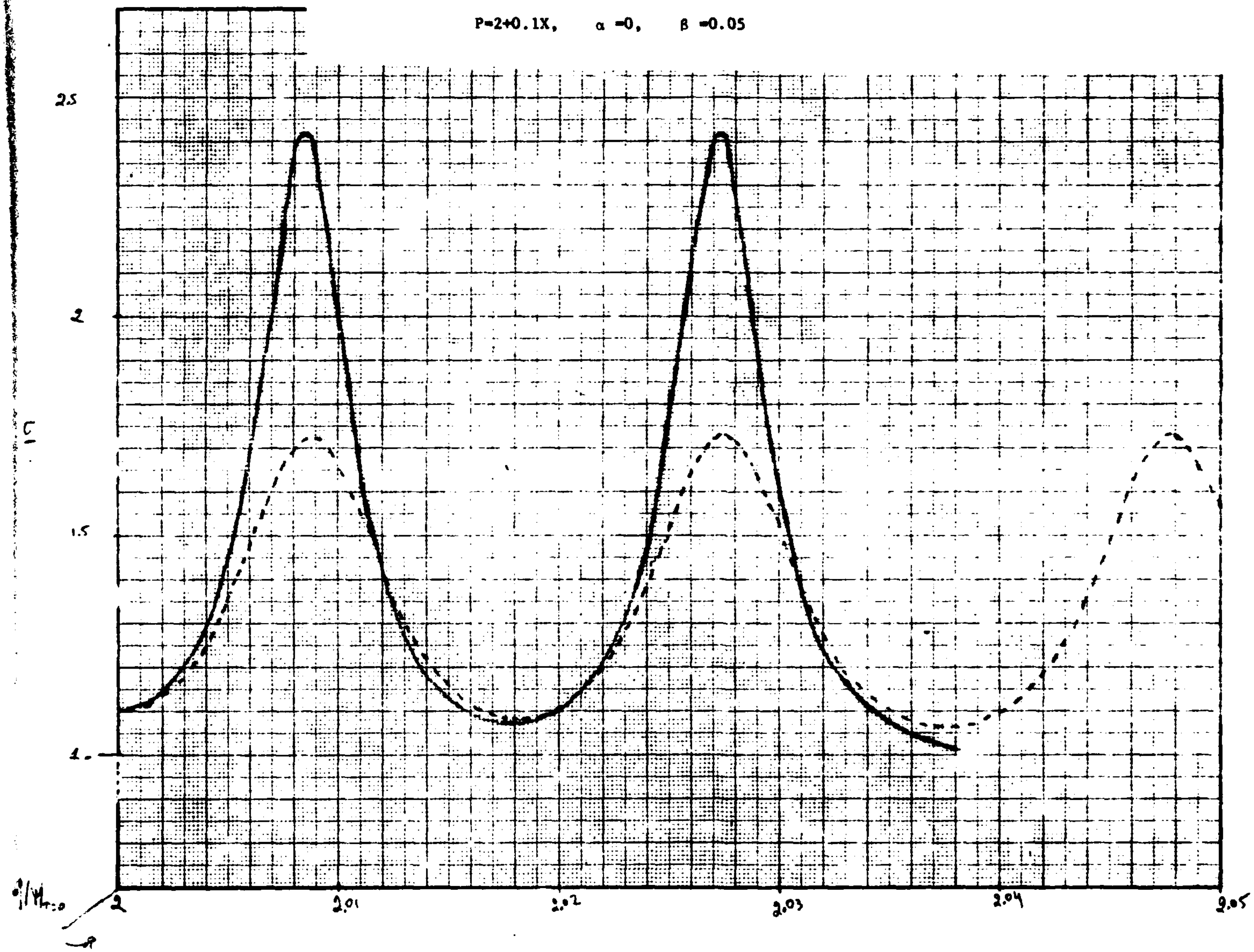
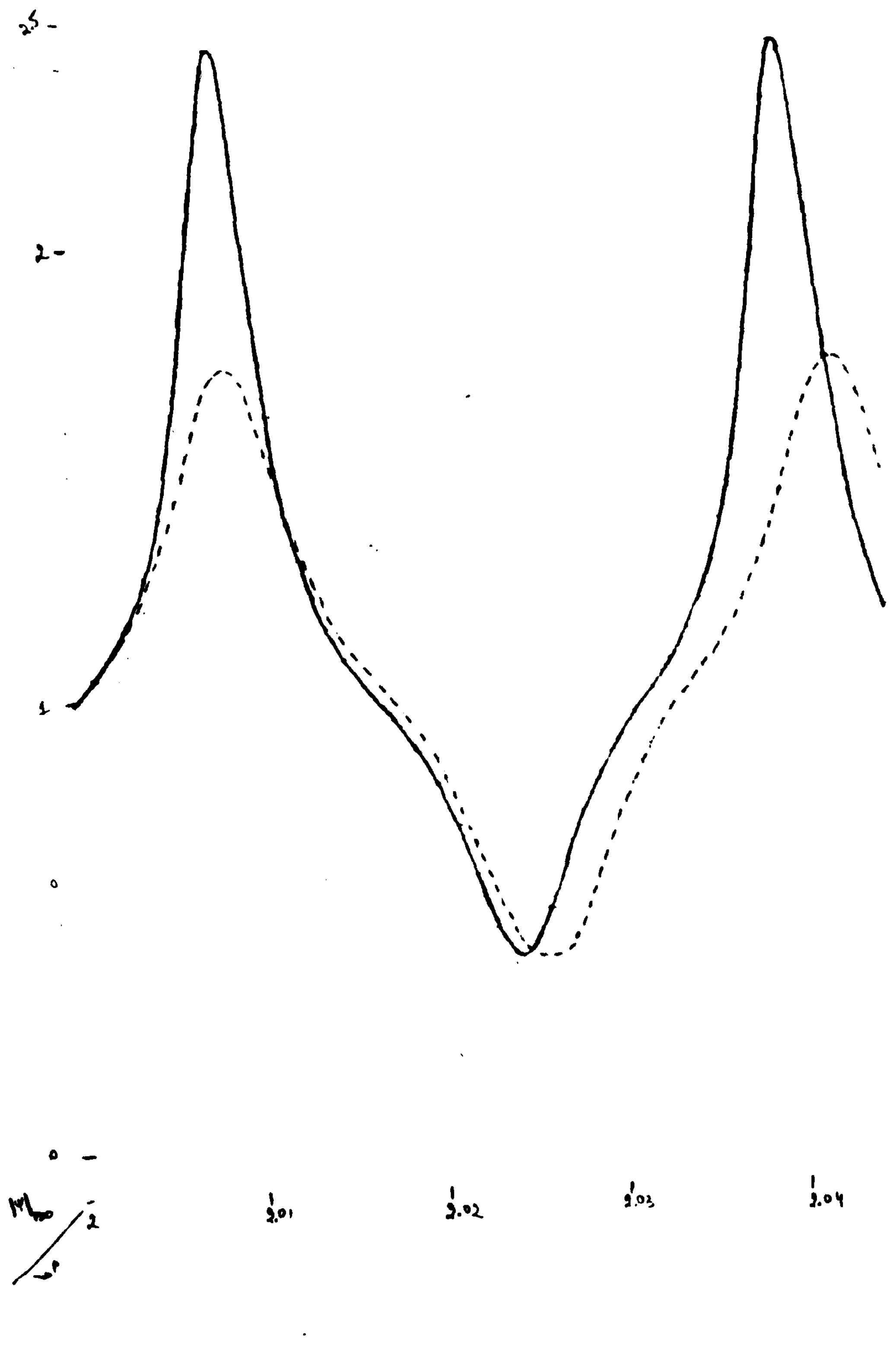


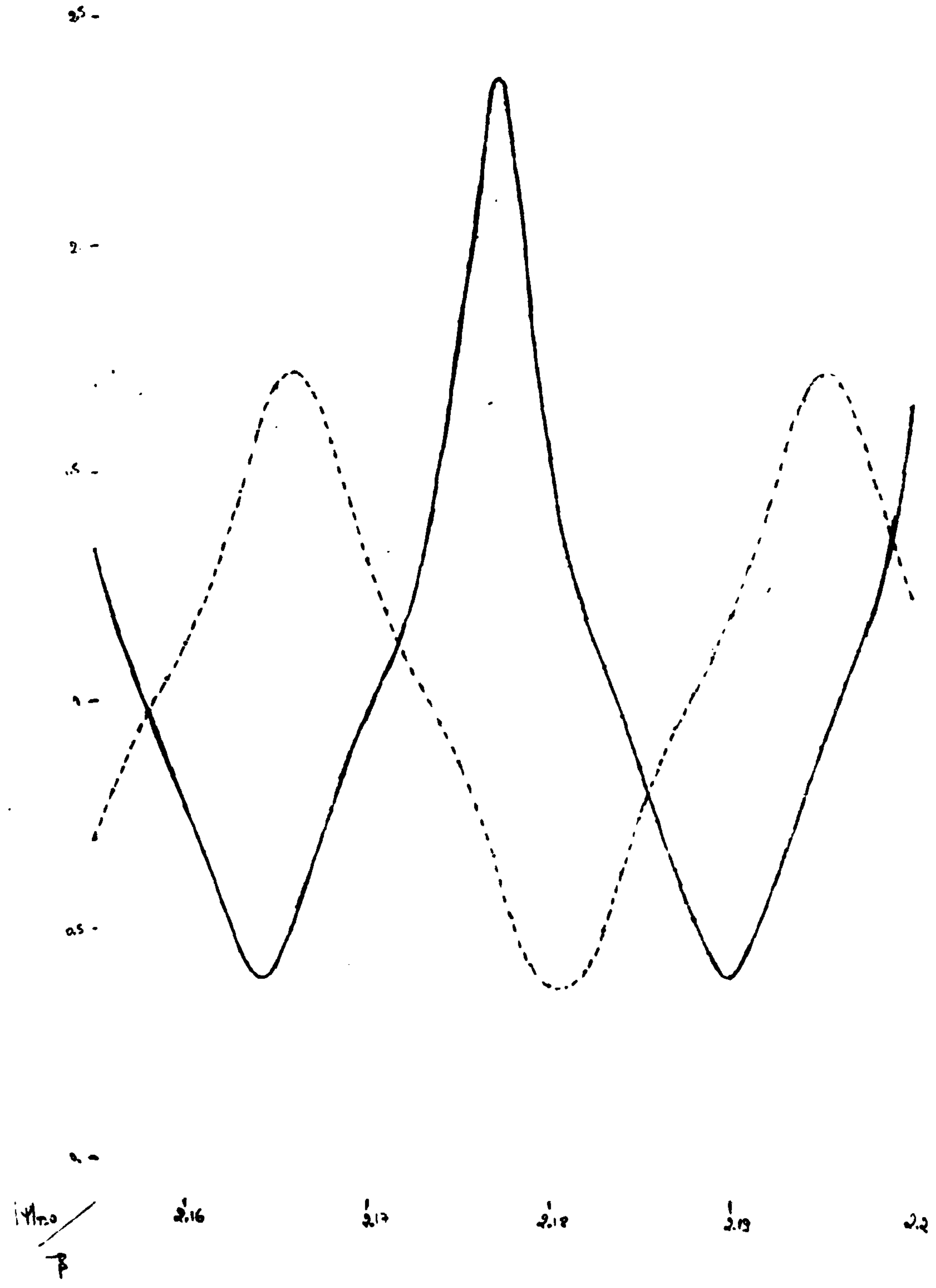
Figure 9: The wave envelope $|\psi(o,x)|$

$P = 2+0.1X$, $\alpha = \pi/2$, $\beta = 0.1$



<u>Figure 10:</u> The wave envelope $|\psi(o,x)|$

$P = 2+0.1X, \quad \alpha = \pi/2, \quad \beta = 0.1$



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